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ANALYSIS OF AN ERROR MASKING SCHEME

FOR PARALLEL COMPUTATIONS

Gerard G. L. Meyer and Howard L. Weinert

Report JHU/EECS-83/19

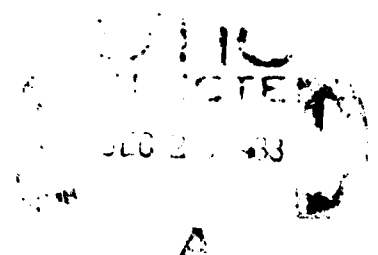DEC 2 8 1983

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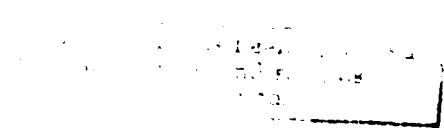
ABSTRACT

The authors

~~We~~ determine the conditions under which an error masking scheme based on strict redundancy can be used to increase our confidence in the results of parallel computations. This study shows that the issues of speed and reliability of parallel processors are interdependent and must be considered jointly at the design stage.

1. INTRODUCTION

In this paper, we continue the investigation, begun in [MEY83], into the use of strict hardware redundancy for increasing confidence in the results of parallel computations. As before, suppose a given problem can be solved in a given time using a cluster of β computing elements, and suppose the following hypothesis holds:

Hypothesis 1:

(i) the input to the cluster is correct,

(ii) each computing element in the cluster has the same probability p of remaining non-faulty during some given "mission time",

(iii) the computing elements fail independently.

If P_C is the probability that the output of the cluster is correct, then Hypothesis 1 implies

$$P_C \geq p^\beta = P_{C,m}. \tag{1}$$

Note that a computing element may be faulty without affecting the cluster output. If P_C is not high enough for our purposes, one way to try to increase our confidence in the cluster output is to accept it only when we think it is correct. This approach is analyzed in [MEY83]. Another approach involves the use of an error masking scheme to directly increase the probability of getting correct results. Such a scheme may be implemented by replicating the original cluster $\alpha-1$ times, sending the original input to all the clusters, and then sending all the cluster outputs to an error masker which, by comparing them, chooses as its out-

put the one most likely to be correct. The masker makes its choice by first partitioning the set of cluster outputs into blocks such that all the outputs in a given block are identical, and then by choosing any of the outputs in the block of maximal cardinality. If more than one block has maximal cardinality, the masker chooses an output at random from among these blocks. The quantity of interest to us is $P_{CM}(\alpha,\beta)$, the probability that the error masker output is correct.

In this paper, we shall not assume that the error masker is always non-faulty; instead, we shall assume that the error masker always produces the desired output whenever a strict majority of its inputs are identical. In other words, we shall adopt the following hypothesis:

Hypothesis 2: Whenever a strict majority of the cluster outputs are identical, the output of the error masker is equal to one of those cluster outputs.

This type of error masking scheme goes back to von Neumann's work in the Fifties [NEU63] on constructing logic devices. His work was extended and applied to relay circuits by Moore and Shannon [MOO56]. See [BAR65] for further work along these lines, and [AVI78] for a survey. Most authors restrict attention to the case $\alpha = 3$ (triple modular redundancy) – see, for example, [LYO62] and [BOU71]. In the context of parallel processing, no complete analysis of the usefulness of error masking, under any reasonable set of assumptions, has been carried out.

The analysis presented in the next section is based on three principles: (i) one should distinguish between hardware faults in a

computing network and incorrect results produced by the network; (ii)
one should assume as little as possible about the fault mechanism; (iii)
one should use only those quantities that have some chance of being
experimentally measured. These considerations rule out, in particular,
the use of a failure probability distribution. They also lead us to a
worst case analysis.

2. ANALYSIS OF THE ERROR MASKING SCHEME

In this section, a worst case analysis of the error masking scheme is presented. Given $\alpha \geq 1$ clusters, each containing β computing elements, let $E_j(\alpha,\beta)$ be the event that exactly j clusters are faulty. A cluster is faulty if and only if at least one of its elements is faulty, and therefore the probability that a cluster is faulty is $1-p^\beta$. As a result,

$$P(E_j(\alpha,\beta)) = \frac{\alpha!}{j!(\alpha-j)!}(1-p^\beta)^j p^{\beta(\alpha-j)}, \quad j = 0,1,\ldots,\alpha. \qquad (2)$$

Let $B_j(\alpha,\beta)$ be the event that exactly j cluster outputs are incorrect. It is clear that if no more than k clusters are faulty, then at most k cluster outputs can be incorrect, and thus

$$\sum_{j=0}^{k} P(B_j(\alpha,\beta)) \geq \sum_{j=0}^{k} P(E_j(\alpha,\beta)). \qquad (3)$$

A lower bound for $P_{CM}(\alpha,\beta)$ will now be derived.

__Lemma__ 1: Under Hypotheses 1 and 2, $P_{CM}(\alpha,\beta) \geq P_{CM,m}(\alpha,\beta)$, where

$$P_{CM,m}(\alpha,\beta) = \sum_{j=0}^{(\alpha-1)/2} \frac{\alpha!}{j!(\alpha-j)!}(1-p^\beta)^j p^{\beta(\alpha-j)}, \quad \alpha \text{ odd}, \qquad (4)$$

and

$$P_{CM,m}(\alpha,\beta) = \sum_{j=0}^{(\alpha-2)/2} \frac{\alpha!}{j!(\alpha-j)!}(1-p^\beta)^j p^{\beta(\alpha-j)}, \quad \alpha \text{ even}. \qquad (5)$$

Proof: By definition

$$P_{CM}(\alpha,\beta) = \sum_{j=0}^{\alpha} P(\text{masker output correct} \mid B_j(\alpha,\beta))P(B_j(\alpha,\beta)),$$

and it is clear from Hypothesis 2 that for $j < \alpha/2$

$$P(\text{masker output correct} \mid B_j(\alpha,\beta)) = 1.$$

Thus, for every $k < \alpha/2$

$$P_{CM}(\alpha,\beta) \geq \sum_{j=0}^{k} P(B_j(\alpha,\beta)) \geq \sum_{j=0}^{k} P(E_j(\alpha,\beta))$$

and the result of the lemma follows from (2).

The behavior of the lower bound $P_{CM,m}(\alpha,\beta)$ as a function of α is characterized in the next four lemmas. First, define q and w via

$$q = p^{\beta}$$

$$w = (1-q)/q$$

Lemma 2: Suppose that Hypotheses 1 and 2 are satisfied. If α is odd, then

$$P_{CM,m}(\alpha+2,\beta) - P_{CM,m}(\alpha,\beta) = \frac{w^{(\alpha+1)/2}(w+1)^{-(\alpha+2)}\alpha!}{((\alpha-1)/2)!((\alpha+1)/2)!}(1-w) .$$

Proof: The definition of w implies that $q = 1/(w+1)$ and we may rewrite (4) as

$$P_{CM,m}(\alpha,\beta) = \sum_{j=0}^{(\alpha-1)/2} \frac{\alpha!}{j!(\alpha-j)!} w^j (w+1)^{-\alpha}$$

$$= (w+1)^{-(\alpha+2)} \alpha! \sum_{j=0}^{(\alpha-1)/2} \frac{1}{j!(\alpha-j)!} w^j (w+1)^2.$$

It follows that

$$P_{CM,m}(\alpha+2,\beta) = (w+1)^{-(\alpha+2)} \alpha! \sum_{j=0}^{(\alpha+1)/2} \frac{(\alpha+2)(\alpha+1)}{j!(\alpha+2-j)!} w^j .$$

Thus,

$$P_{CM,m}(\alpha+2,\beta) - P_{CM,m}(\alpha,\beta) = (w+1)^{-(\alpha+2)} \alpha! \sum_{j=0}^{(\alpha+3)/2} \gamma_j w^j , \qquad (6)$$

where

$$\gamma_0 = \frac{(\alpha+2)(\alpha+1)}{(\alpha+2)!} - \frac{1}{\alpha!} ,$$

$$\gamma_1 = \frac{(\alpha+2)(\alpha+1)}{(\alpha+1)!} - \frac{2}{\alpha!} - \frac{1}{(\alpha-1)!} ,$$

$$\gamma_j = \frac{(\alpha+2)(\alpha+1)}{j!(\alpha+2-j)!} - \frac{1}{(j-2)!(\alpha-j+2)!} - \frac{2}{(j-1)!(\alpha-j+1)!} - \frac{1}{j!(\alpha-j)!} ,$$

for $j = 2, 3, \ldots, (\alpha-1)/2$,

$$\gamma_{(\alpha+1)/2} = \frac{(\alpha+2)(\alpha+1)}{((\alpha+1)/2)!((\alpha+3)/2)!} - \frac{1}{((\alpha-3)/2)!((\alpha+3)/2)!} - \frac{2}{((\alpha-1)/2)!((\alpha+1)/2)!} ,$$

and

$$\gamma_{(\alpha+3)/2} = - \frac{1}{((\alpha-1)/2)!((\alpha+1)/2)!} .$$

It is not difficult to verify that

$$\gamma_j = 0, \quad j = 0, 1, 2, \ldots, (\alpha-1)/2,$$

and

$$\gamma_{(\alpha+1)/2} = \frac{1}{((\alpha-1)/2)!((\alpha+1)/2)!} \; .$$

Substituting into (6) yields the desired result.

Lemma 3: Suppose that Hypotheses 1 and 2 are satisfied. If α is even, then

$$P_{CM,m}(\alpha+2,\beta) - P_{CM,m}(\alpha,\beta) = \frac{w^{\alpha/2}(w+1)^{-(\alpha+2)}\alpha!}{((\alpha-2)/2)!((\alpha+2)/2)!}((\alpha+2)/\alpha-w) \; .$$

Proof: Rewrite (5) as

$$P_{CM,m}(\alpha,\beta) = \sum_{j=0}^{(\alpha-2)/2} \frac{\alpha!}{j!(\alpha-j)!} \; w^j(w+1)^{-\alpha}$$

$$= (w+1)^{-(\alpha+2)}\alpha! \sum_{j=0}^{(\alpha-2)/2} \frac{1}{j!(\alpha-j)!} \; w^j(w+1)^2 .$$

It follows that

$$P_{CM,m}(\alpha+2,\beta) = (w+1)^{-(\alpha+2)}\alpha! \sum_{j=0}^{\alpha/2} \frac{(\alpha+2)(\alpha+1)}{j!(\alpha+2-j)!} \; w^j \; .$$

Thus,

$$P_{CM,m}(\alpha+2,\beta) - P_{CM,m}(\alpha,\beta) = (w+1)^{-(\alpha+2)}\alpha! \sum_{j=0}^{(\alpha+2)/2} \gamma_j w^j, \qquad (7)$$

where

$$\gamma_0 = \frac{(\alpha+2)(\alpha+1)}{(\alpha+2)!} - \frac{1}{\alpha!} \; ,$$

$$\gamma_1 = \frac{(\alpha+2)(\alpha+1)}{(\alpha+1)!} - \frac{2}{\alpha!} - \frac{1}{(\alpha-1)!},$$

$$\gamma_j = \frac{(\alpha+2)(\alpha+1)}{j!(\alpha+2-j)!} - \frac{1}{(j-2)!(\alpha-j+2)!} - \frac{2}{(j-1)!(\alpha-j+1)!} - \frac{1}{j!(\alpha-j)!},$$

for $j = 2, 3, \ldots, (\alpha-2)/2$,

$$\gamma_{\alpha/2} = \frac{(\alpha+2)(\alpha+1)}{(\alpha/2)!((\alpha+4)/2)!} - \frac{1}{((\alpha-4)/2)!((\alpha+4)/2)!} - \frac{2}{((\alpha-2)/2)!((\alpha+2)/2)!},$$

and

$$\gamma_{(\alpha+2)/2} = - \frac{1}{((\alpha-2)/2)!((\alpha+2)/2)!}.$$

It is not difficult to verify that

$$\gamma_j = 0, \quad j = 0, 1, 2, \ldots, (\alpha-2)/2,$$

and

$$\gamma_{\alpha/2} = \frac{((\alpha+2)/\alpha)}{((\alpha-2)/2)!((\alpha+2)/2)!}.$$

Substituting into (7) yields the *desired result*.

Lemma 4: Suppose that Hypotheses 1 and 2 are satisfied. If α is odd, then

$$P_{CM,m}(\alpha+1,\beta) - P_{CM,m}(\alpha,\beta) = - \frac{w^{(\alpha+1)/2}(w+1)^{-(\alpha+1)}\alpha!}{((\alpha-1)/2)!((\alpha+1)/2)!}.$$

Proof: Since α is odd, we may rewrite (4) and (5) as

$$P_{CM,m}(\alpha,\beta) = (w+1)^{-(\alpha+1)}\alpha! \sum_{j=0}^{(\alpha-1)/2} \frac{1}{j!(\alpha-j)!} w^j(w+1),$$

and

$$P_{CM,m}(\alpha+1,\beta) = (w+1)^{-(\alpha+1)}\alpha! \sum_{j=0}^{(\alpha-1)/2} \frac{(\alpha+1)}{j!(\alpha+1-j)!} w^j$$

Thus,

$$P_{CM,m}(\alpha+1,\beta) - P_{CM,m}(\alpha,\beta) = (w+1)^{-(\alpha+1)}\alpha! \sum_{j=0}^{(\alpha+1)/2} \gamma_j w^j, \qquad (8)$$

where

$$\gamma_0 = \frac{(\alpha+1)}{(\alpha+1)!} - \frac{1}{\alpha!} ,$$

$$\gamma_j = \frac{(\alpha+1)}{j!(\alpha+1-j)!} - \frac{1}{(j-1)!(\alpha-j+1)!} - \frac{1}{j!(\alpha-j)!},$$

for $j = 1, 2, 3, \ldots, (\alpha-1)/2$, and

$$\gamma_{(\alpha+1)/2} = - \frac{1}{((\alpha-1)/2)!((\alpha+1)/2)!}.$$

It is not difficult to verify that

$$\gamma_j = 0, \quad j = 0, 1, 2, \ldots, (\alpha-1)/2,$$

and thus (8) reduces to the desired result.

Lemma 5: Suppose that Hypotheses 1 and 2 are satisfied. If $p^\beta > 0.5$, then $P_{CM,m}(\alpha,\beta)$ converges to one as α goes to infinity; if $p^\beta = 0.5$, then $P_{CM,m}(\alpha,\beta)$ converges to 0.5 as α goes to infinity; and if $p^\beta < 0.5$, then $P_{CM,m}(\alpha,\beta)$ converges to zero as α goes to infinity.

Proof: Using the well known Gaussian approximation to the binomial distribution, we have

$$\lim_{\alpha\to\infty} P_{CM,m}(\alpha,\beta) = \lim_{\alpha\to\infty} (2\pi)^{-0.5} \int_{t_1(\alpha)}^{t_2(\alpha)} e^{-0.5t^2} dt ,$$

where

$$t_1(\alpha) = -\alpha(1-q)((\alpha q(1-q))^{-0.5} ,$$

$$t_2(\alpha) = (\alpha(q-0.5)-0.5)((\alpha q(1-q))^{-0.5} , \; \alpha \text{ odd,}$$

$$t_2(\alpha) = \alpha(q-0.5)-1)((\alpha q(1-q))^{-0.5} , \; \alpha \text{ even .}$$

It is clear that

$$\lim_{\alpha \to \infty} t_1(\alpha) = -\infty ,$$

$$\lim_{\alpha \to \infty} t_2(\alpha) = +\infty , \; q > 0.5 ,$$

$$\lim_{\alpha \to \infty} t_2(\alpha) = 0 , \; q = 0.5 ,$$

$$\lim_{\alpha \to \infty} t_2(\alpha) = -\infty , \; q < 0.5 ,$$

and thus

$$\lim_{\alpha \to \infty} (2\pi)^{-0.5} \int_{t_1(\alpha)}^{t_2(\alpha)} e^{-0.5t^2} dt = 1 , \; q > 0.5 ,$$

$$\lim_{\alpha \to \infty} (2\pi)^{-0.5} \int_{t_1(\alpha)}^{t_2(\alpha)} e^{-0.5t^2} dt = 0.5 , \; q = 0.5 ,$$

$$\lim_{\alpha \to \infty} (2\pi)^{-0.5} \int_{t_1(\alpha)}^{t_2(\alpha)} e^{-0.5t^2} dt = 0 , \; q < 0.5 .$$

Lemma 2 implies that if α is odd, $P_{CM,m}(\alpha,\beta)$ is a strictly increasing function of α when $p^\beta > 0.5$, $P_{CM,m}(\alpha,\beta)$ is constant when $p^\beta = 0.5$, and $P_{CM,m}(\alpha,\beta)$ is a strictly decreasing function of α when $p^\beta < 0.5$.

Lemma 3 implies that if α is even, $P_{CM,m}(\alpha+2,\beta) > P_{CM,m}(\alpha,\beta)$ when $p^\beta > \alpha/(2(\alpha+1))$, $P_{CM,m}(\alpha+2,\beta) = P_{CM,m}(\alpha,\beta)$ when $p^\beta = \alpha/(2(\alpha+1))$, and

$P_{CM,m}(\alpha+2,\beta) < P_{CM,m}(\alpha,\beta)$ when $p^\beta < \alpha/(2(\alpha+1))$. It follows that if α is even, $P_{CM,m}(\alpha,\beta)$ is a strictly increasing function of α when $p^\beta \geq 0.5$; $P_{CM,m}(\alpha,\beta)$ first increases, reaches a maximum value that depends on p^β, and then decreases when p^β is in the interval $(1/3,1/2)$; $P_{CM,m}(4,\beta) = P_{CM,m}(2,\beta)$ and $P_{CM,m}(\alpha,\beta)$ is a strictly decreasing function of α for $\alpha \geq 4$ when $p^\beta = 1/3$; and finally, $P_{CM,m}(\alpha,\beta)$ is a strictly decreasing function of α when $p^\beta < 1/3$.

Lemma 4 implies that if α is odd, $P_{CM,m}(\alpha+1,\beta) < P_{CM,m}(\alpha,\beta)$ for all values of p^β. In other words, it is never advantageous to choose an even α.

Under what conditions, then, can this error masking scheme be used to increase our confidence in the results of parallel computations? In light of Lemmas 2, 4, and 5 and the fact that $P_{CM,m}(1,\beta) = P_{C,m}$, error masking (with an odd number of clusters) can be used to guarantee that $P_{CM}(\alpha,\beta) > P_{C,m}$ if and only if $p^\beta > 0.5$, in which case we can ensure that $P_{CM}(\alpha,\beta)$ is as close to one as we wish by choosing α sufficiently large.

We have just seen that even when $p^\beta > 0.5$, there is no way to ensure that $P_{CM}(\alpha,\beta) = 1$ using a finite number of computing elements. However, if one does not adhere to the principles discussed in the introduction, and is willing to make stronger, more optimistic, assumptions, then it is possible to obtain $P_{CM}(\alpha,\beta) = 1$ with a finite α. For example, if, in addition to Hypotheses 1 and 2, we assume that the number of faulty computing elements is at most ζ, then the choice of any

$\alpha \geq 2\xi + 1$ guarantees that the number of correct cluster outputs is strictly greater than the number of incorrect cluster outputs. In this case, the masker always chooses a correct output, and therefore $P_{CM}(\alpha,\beta)$ = 1. Alternatively, if in addition to Hypothesis 1, we assume that (i) whenever there is at most one block of cardinality greater than one, the masker chooses an output from that block, (ii) the number of faulty computing elements is at most ξ, and (iii) any two incorrect cluster outputs must be distinct, then any choice of $\alpha \geq \xi + 2$ ensures that $P_{CM}(\alpha,\beta)$ = 1.

3. EFFICIENT COMPUTING NETWORK DESIGN

Suppose that that it is possible to solve a given problem in a given time using a cluster of β computing elements of a given type, and assume that the basic reliability p of this type of computing element is known. Furthermore, suppose that we want to design a computing network consisting of a such clusters and an error masker, so that the masker output is correct with probability at least Θ ($0 < \Theta < 1$). To meet this requirement, it is sufficient, in view of the results of the previous section, to ensure that the following inequality is satisfied for some odd integer $a \geq 1$:

$$\sum_{j=0}^{(a-1)/2} \frac{a!}{j!(a-j)!}(1-p^{\beta})^j p^{\beta(a-j)} \geq \Theta. \qquad (9)$$

Given p, β and Θ, it may or may not be possible to satisfy (9). If (9) can be satisfied, the most efficient design is obtained when a is chosen to be the smallest odd integer a_{Θ} that satisfies (9). In order to analyze the feasibility and efficiency issues, we partition the set of all pairs (Θ, p^{β}) into disjoint subsets V_0, V_1 and V_M as follows (see Figure 1):

$$V_0 = \{(\Theta, p^{\beta}) \mid p^{\beta} < \Theta \leq 0.5 \} \cup \{(\Theta, p^{\beta}) \mid p^{\beta} \leq 0.5 < \Theta \},$$

$$V_1 = \{(\Theta, p^{\beta}) \mid p^{\beta} \geq \Theta \},$$

$$V_M = \{(\Theta, p^{\beta}) \mid 0.5 < p^{\beta} < \Theta \}.$$

The following lemma is a direct consequence of the results of the preceding section.

Lemma 6: If (Θ, p^β) is in V_0, then (9) cannot be satisfied. If (Θ, p^β) is in V_1, then (9) can be satisfied and $\alpha_* = 1$. If (Θ, p^β) is in V_M, then (9) can be satisfied and $\alpha_* \geq 3$.

We know that when the pair (Θ, p^β) is in V_M, the error masking scheme may be used to increase our confidence in the output. The amount of necessary replication, given by the value of α_*, depends on the precise location of the point (Θ, p^β) in the region V_M. Figure 2 shows the decomposition of V_M into subregions for which α_* equals 3, 5, 7,..., etc. These regions are relatively narrow, and become smaller as α_* increases. It follows that small changes in p^β may lead to large changes in α_*. We now present an example that illustrates this situation.

Example: Suppose that we want to ensure that the probability that the output of the masking scheme is correct is at least 0.95. This constraint will automatically be met if (9) is satisfied with $\Theta = 0.95$. Thus:

if $0.950 \leq p^\beta$, then $\alpha_* = 1$, and no replication and error masking is needed,

if $0.865 \leq p^\beta < 0.950$, we need $\alpha_* = 3$,

if $0.811 \leq p^\beta < 0.865$, we need $\alpha_* = 5$,

if $0.775 \leq p^\beta < 0.811$, we need $\alpha_* = 7$,

if $0.749 \leq p^\beta < 0.775$, we need $\alpha_* = 9$,

if $0.729 \leq p^\beta < 0.749$, we need $\alpha_* = 11$,

and so forth. It is clear that the necessary amount of replication increases quite rapidly with decreasing p^β. Suppose now that we a priori fix α to be equal to 3 (the triple modular redundancy case). Then, in order to guarantee that $P_{CM}(3,\beta) > 0.95$, we must have $p^\beta \geq 0.865$. For a given p (that is, for a given type of computing element), this translates into an upper bound on the number β of computing elements that can be used in each cluster; in particular,

$$\beta \leq \beta_* = \frac{\log 0.865}{\log p}$$

where x is defined as the largest integer less than or equal to x. Thus, if $p = 0.99$, $\beta_* = 14$; if $p = 0.95$, $\beta_* = 2$; and if $p = 0.90$, $\beta_* = 1$. A limit on β is clearly a limit on the computational speed of the network, thus emphasizing the fact that the speed and reliability of parallel processors are interdependent. Finally, it is interesting to ask what minimum computing element reliability p is required in order to use a large number of elements in each cluster, while ensuring that $P_{CM}(3,\beta) > 0.95$. Since we need $p^\beta \geq 0.865$,

$$p \geq p_* = (0.865)^{1/\beta}.$$

Thus, if $\beta = 256$, $p_* = 0.9994$; if $\beta = 1024$, $p_* = 0.9998$; and if $\beta = 2048$, $p_* = 0.9999$.

4. CONCLUSION

The results of this paper show that, when reasonable assumptions are made, strict hardware redundancy is of limited value in meeting parallel processing reliability constraints. If the reliability of the basic computing element is not high enough, it is impossible to achieve the desired network reliability using strict redundancy (Figure 1, Region V_0). If the reliability of the basic computing element is sufficiently high, no redundancy is needed to achieve the desired network reliability (Figure 1, Region V_1). The remaining case (Figure 1, Region V_M) is the only one in which strict redundancy is of use. However, the amount of replication needed to satisfy the reliability constraint may be quite large, as illustrated in Figure 2.

In view of the preceding discussion, it is clear that an alternative approach to meeting reliability constraints, without massive amounts of hardware, is desirable. The authors are presently working on such an approach in the context of signal processing. The basic idea consists in taking into account the structure of the problem to be solved and in exploiting any inherent redundancy in the input data [MEY82].

REFERENCES

[AVI78]   Avizienis, A., Fault-Tolerance: The Survival Attribute of Digital Systems, *Proc*. *IEEE*, Vol. 66, No. 10, October 1978, pp. 1109-1125.

[BAR65]   Barlow, R. E. and Proschan, F., *Mathematical Theory of Reliability*, John Wiley, New York, New York, 1965.

[BOU71]   Bouricius, W. G. *et al*, Reliability Modeling for Fault-Tolerant Computers, *IEEE Trans*. *Computers*, Vol. 20, 1971, pp. 1306-1311.

[LYO62]   Lyons, R. E. and Vanderkulk, W., The Use of Triple-Modular Redundancy to Improve Computer Reliability, *IBM J*. *Res*. *Devel*., Vol. 6, 1962, pp. 200-209.

[MEY82]   Meyer, G. G. L. and Weinert, H. L., Parallel Algorithms and Computational Structures for Linear Estimation Problems, *Proc*. *ONR Workshop on Signal Processing in the Ocean Environment*, to be published by Marcel Dekker.

[MEY83]   Meyer, G. G. L. and Weinert, H. L., Analysis of an Error Detection Scheme for Parallel Computations, Report JHU/EECS-83/14, Department of Electrical Engineering and Computer Science, The Johns Hopkins University, Baltimore, Maryland.

[MOO56]   Moore, E. F. and Shannon, C. E., Reliable Circuits Using Less Reliable Relays, *J*. *Franklin Inst*., Vol. 262, 1956, pp. 191-208

and pp. 281-297.

[NEU63] von Neumann, J., <u>Collected Works</u>, Taub, A. H., ed., Vol. 5, MacMillan, New York, New York, 1963.
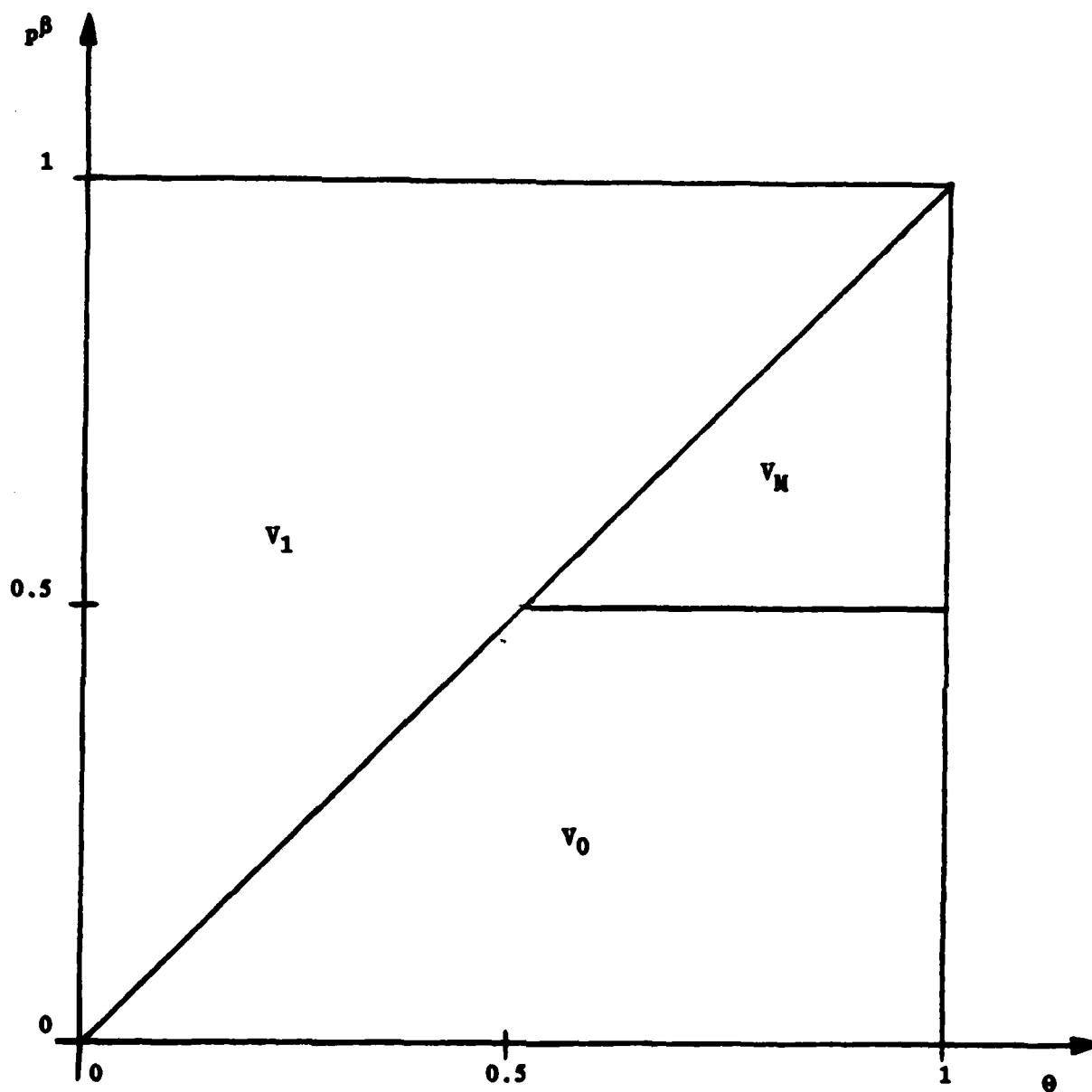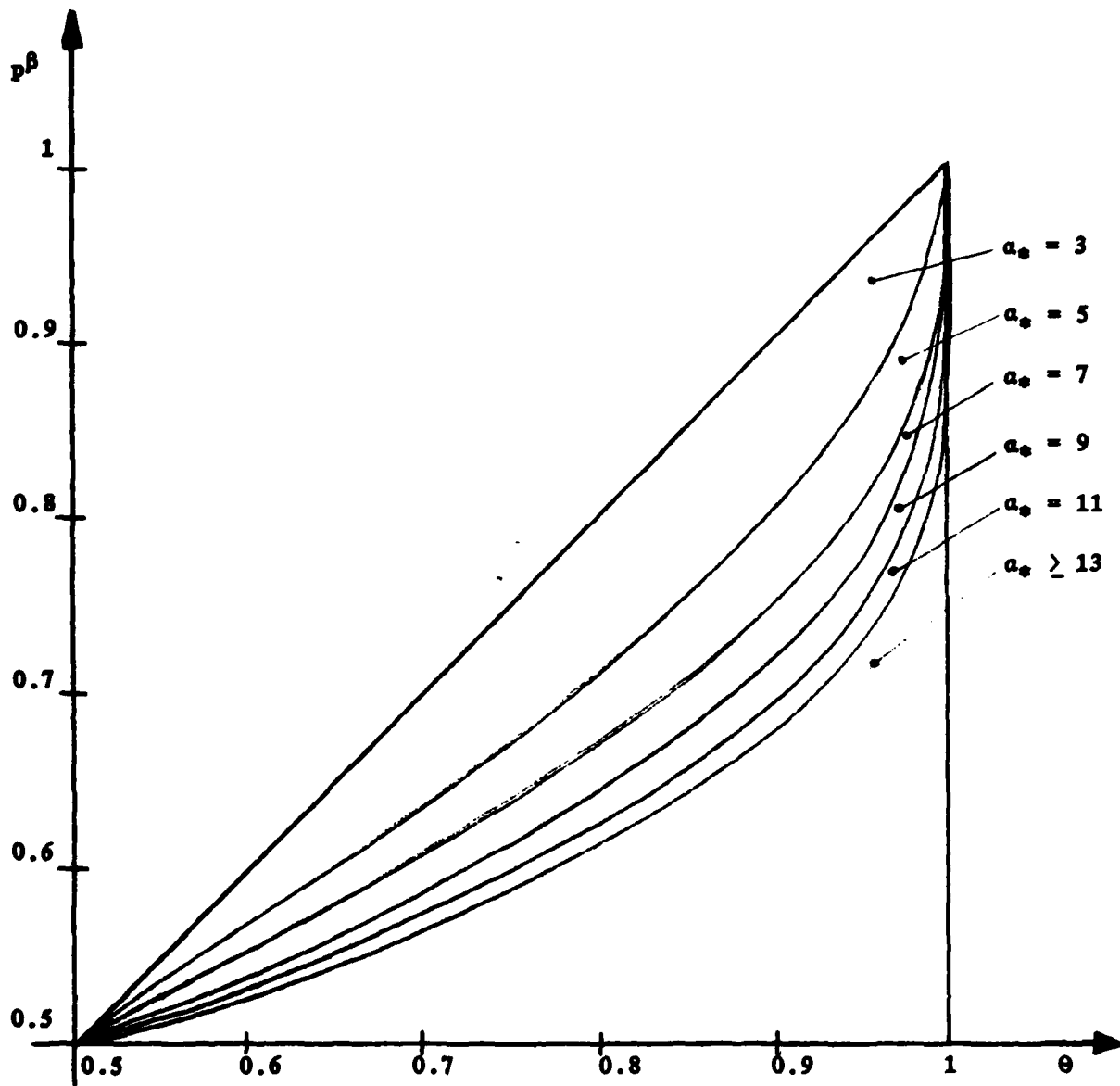
Figure 1: Feasibility Regions for Masking

Figure 2: Decomposition of Region V_M

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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

We determine the conditions under which an error masking scheme based on strict redundancy can be used to increase our confidence in the results of parallel computations. This study shows that the issues of speed and reliability of parallel processors are interdependent and must be considered jointly at the design stage.

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